2021
MATHEMATICS

Full marks: 100 Time: 3 hours

General instructions:

- *i) Approximately 15 minutes is allotted to read the question paper and revise the answers.*
- *ii) The question paper consists of 26 questions. All questions are compulsory.*
- *iii) Marks are indicated against each question.*
- *iv) Internal choice has been provided in some questions.*
- *v) Use of simple calculators (non-scientific and non-programmable) only is permitted.*

N.B: Check that all pages of the question paper is complete as indicated on the top left side.

Section – A

1. **Choose the correct answer from the given alternatives:**

(a) If $f: \mathbf{R} \rightarrow \mathbf{R}$ is defined as $f(x) = x^4$, then **1**

(i) f is one-one onto (ii) f is many-one onto
(iii) f is one-one but not onto (iv) f is neither one-one nor onto

(b) The principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is **1**

(i) $\frac{\pi}{6}$ (ii) $\frac{\pi}{4}$ (iii) $\frac{\pi}{3}$ (iv) $\frac{\pi}{2}$

(c) $A = [\, a_{ij} \,]_{m \times n}$ is a square matrix, if **1**

(i) $m < n$ (ii) $m > n$ (iii) $m = n$ (iv) none of these

(d) If $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then $\frac{dy}{dx}$ is equal to **1**

(i) $\frac{2}{1-x^2}$ (ii) $\frac{2}{1+x^2}$ (iii) $\frac{-2}{1+x^2}$ (iv) $\frac{1}{1+x^2}$

(e) If $x = a\cos\theta$, $y = b\cos\theta$, then $\frac{dy}{dx}$ is equal to **1**

(i) $\frac{a}{b}$ (ii) $\frac{-a}{b}$ (iii) $\frac{b}{a}$ (iv) $\frac{-b}{a}$

(f) $\int \frac{xdx}{(x-1)(x-2)}$ is equal to **1**

(i) $\log\left|\frac{(x-1)^2}{x-2}\right|+C$ (ii) $\log\left|\frac{(x-2)^2}{x-1}\right|+C$

(iii) $\log\left|\left(\frac{x-1}{x-2}\right)^2\right|+C$ (iv) $\log\left|(x-1)(x-2)\right|+C$

(g) $\int \frac{dx}{x^2+2x+2}$ is equal to **1**

(i) $x\tan^{-1}(x+1)+C$ (ii) $\tan^{-1}(x+1)+C$

(iii) $(x+1)\tan^{-1}x+C$ (iv) $\tan^{-1}x+C$

(h) If $\vec{a}$ is a nonzero vector of magnitude a and λ a nonzero scalar, then $\lambda\vec{a}$ is unit vector if **1**

(i) $\lambda = 1$ (ii) $\lambda = -1$ (iii) $a = |\lambda|$ (iv) $a = \frac{1}{|\lambda|}$

(i) Let the vectors $\vec{a}$ and $\vec{b}$ be such that $\left|\vec{a}\right| = 3$ and $\left|\vec{b}\right| = \frac{\sqrt{2}}{3}$, then $\vec{a}\times\vec{b}$ is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$ is **1**

(i) $\frac{\pi}{6}$ (ii) $\frac{\pi}{4}$ (iii) $\frac{\pi}{3}$ (iv) $\frac{\pi}{2}$

(j) The direction cosines of x- axis are **1**

(i) 0, 0, 1 (ii) 0, 1, 0 (iii) 1, 0, 0 (iv) 0, 0, 0

Section – B

2. Find $g\circ f$ and $f\circ g$ if $f(x)=8x^3$ and $g(x)=x^{\frac{1}{3}}$ **2**

3. Find the value of $\tan^{-1}(1)+\cos^{-1}\left(\frac{-1}{2}\right)+\sin^{-1}\left(\frac{-1}{2}\right)$ **2**

4. Construct a 2 × 2 matrix, A = [a_{ij}]$_{m\times n}$, whose elements are given by, $a_{ij} = \frac{(i+j)^2}{2}$ **2**

5. Find $\frac{dy}{dx}$ if $x^2+xy+y^2=100$ **2**

6. The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm. **2**

7. Evaluate $\int \frac{e^{\tan^{-1} x}}{1 + x^2}\, dx$ **2**

8. Find the general solution of $\frac{dy}{dx} = \left(1+x^2\right)\left(1+y^2\right)$ **2**

9. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ **2**

10. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\left|\vec{a}\right| = 5, \left|\vec{b}\right| = 12, \left|\vec{c}\right| = 13$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ **2**

11. If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, find P(not A and not B) **2**

Section – C

12. **a.** Consider $f: \mathbf{R} \to \mathbf{R}$ given by $f(x) = 4x+3$. Show that f is invertible & find its inverse

Or **4**

b. Write the domain and range of the following inverse trigonometric functions: (i) $\sin^{-1} x$ (ii) $\cos^{-1} x$ (iii) $\tan^{-1} x$ (iv) $\cot^{-1} x$

13. For the matrix $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$, show that $A^2 - 4A + 7I = O$. Hence, find A^{-1} **4**

14. **a.** If $y = \left(\tan^{-1} x\right)^2$, show that $\left(x^2+1\right)^2 y_2 + 2x\left(x^2+1\right)y_1 = 2$

Or **4**

b. If $x = \sqrt{a^{\sin^{-1} t}}$, $y = \sqrt{a^{\cos^{-1} t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$

15. **a.** Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to the x-axis.

Or **4**

b. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$

16. **a.** Evaluate: $\int \frac{2x}{(x^2+1)(x^2+3)}\,dx$

Or **4**

b. Evaluate: $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$

17. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x}\,dx$ **4**

18. Show that the differential equation: $(x^2 + xy)dy = (x^2 + y^2)dx$ is homogeneous and solve it. **4**

19. Find the equation of the plane through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and the point (2, 2, 1) **4**

20. Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade. What is the probability that the student is a hosteller? **4**

21. **a.** A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Determine: (i) k, (ii) P(X < 3), (iii) P(X > 6), (iv) P(0 < X < 3)

Or **4**

b. The random variable X has a probability distribution P(X) of the following form, where k is some number.

$$P(X) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

(i) Determine the value of k. (ii) Find P(X < 2), P(X $\leq$ 2), P(X $\geq$ 2)

Section – D

22. Solve the following system of linear equations using matrix method:

a.
$$\begin{aligned} x - y + z &= 4 \\ 2x + y - 3z &= 0 \\ x + y + z &= 2 \end{aligned}$$

Or **6**

b.
$$\begin{aligned} 2x + 3y + 3z &= 5 \\ x - 2y + z &= -4 \\ 3x - y - 2z &= 3 \end{aligned}$$

23. **a.** Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

Or **6**

b. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$

24. **a.** Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$

Or **6**

b. Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$

25. **a.** Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

Or **6**

b. Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$

26. **a.** A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of ₹7 and screws B at a profit of ₹10. Assuming that he can sell all the screws he manufactures, how many packages

of each type should the factory owner produce in a day in order to maximize his profit? Determine the maximum profit.

Or **6**

b. A merchant plans to sell two types of personal computers – a desktop model and a portable model that will cost ₹25000 and ₹40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than ₹70 lakhs and if his profit on the desktop model is ₹4500 and on portable model is ₹5000.

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