

2022
MATHEMATICS

Full marks: 80

Time: 3 hours

General instructions:

- i) Approximately 15 minutes is allotted to read the question paper and revise the answers.
- ii) The question paper consists of 21 questions. All questions are compulsory.
- iii) Marks are indicated against each question.
- iv) Internal choice has been provided in some questions.
- v) Use of simple calculators (non-scientific and non-programmable) only is permitted.

N.B: Check to ensure that all pages of the question paper is complete as indicated on the top left side.

Section – A**1. Choose the correct answer from the given alternatives:**

- (a) If $B = \{x : x \text{ is a natural number less than } 6\}$, then $n(B)$ is **1**
 (i) 4 (ii) 5 (iii) 6 (iv) 8
- (b) The value of $\cos 405^\circ$ is equal to **1**
 (i) $-\frac{1}{\sqrt{2}}$ (ii) $\frac{1}{\sqrt{2}}$ (iii) $\frac{\sqrt{3}}{2}$ (iv) $-\frac{\sqrt{3}}{2}$
- (c) The multiplicative inverse of $-i$ is **1**
 (i) $-i$ (ii) $\frac{-1}{i}$ (iii) i (iv) $\frac{1}{i}$
- (d) If ${}^5P_r = 2^6 P_{r-1}$, then the value of r is equal to **1**
 (i) 3 (ii) 4 (iii) 5 (iv) 6
- (e) The co-efficient of x^3 in the expansion of $\left(\frac{x}{3} + \frac{1}{x}\right)^5$ is equal to **1**
 (i) $\frac{10}{27}$ (ii) $\frac{5}{81}$ (iii) $\frac{10}{9}$ (iv) $\frac{5}{3}$
- (f) The value of x , where the numbers $-\frac{2}{7}$, x , $-\frac{7}{2}$ are in G.P is **1**
 (i) ± 1 (ii) ± 2 (iii) ± 3 (iv) ± 4

- (g) If a line through the points $(-2, 6)$ and $(4, 8)$ is perpendicular to the line through the points $(8, 12)$ and $(x, 24)$, then 1
(i) $x = 1$ (ii) $x = 2$ (iii) $x = 3$ (iv) $x = 4$
- (h) The contrapositive of the statement $p \Rightarrow q$ is 1
(i) $\sim q \Rightarrow \sim p$ (ii) $\sim p \Rightarrow q$ (iii) $\sim q \Rightarrow p$ (iv) $q \Rightarrow p$

Section – B

2. In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages? 2
3. Find the domain and range of the functions $f(x) = -|x|$. 2
4. Solve the linear inequality: $2(2x + 3) - 10 < 6(x - 2)$ 2
5. Find the 4th term in the expansion of $(x - 2y)^{12}$. 2
6. Given that P $(3, 2, -4)$, Q $(5, 4, -6)$ and R $(9, 8, -10)$ are collinear. Find the ratio in which Q divides PR. 2
7. Evaluate $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$ 2
8. Find the derivative of $x^{-3}(5 + 3x)$. 2
9. Write the contrapositive and converse of the statement:
If x is a prime number, then x is odd 2

Section – C

10. a. Prove that $\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$

Or 4

b. For any ΔABC , prove that $\frac{\sin(B - C)}{\sin(B + C)} = \frac{b^2 - c^2}{a^2}$.

11. Prove that $\sin 2x + 2 \sin 4x + \sin 6x = 4 \cos^2 x \sin 4x$. 4

12. Using the Principle of Mathematical Induction ,prove that for all $n \in \mathbb{N}$

a. $1.2.3 + 2.3.4 + \dots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$

Or

b. $3^{2n+2} - 8n - 9$ is divisible by 8

4

13. Convert the complex number $-1 + i$ in polar form.

4

14. Solve the following system of inequalities graphically:

$4x + 3y \leq 60, y \geq 2x, x \geq 3, x, y \geq 0$

4

15. a. Find the number of 4 digit numbers that can be formed using the digits 1, 2, 3, 4, 5 if no digit is repeated. How many of these will be even?

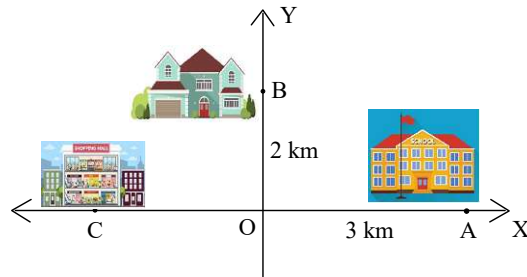
Or

b. In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of eleven must include exactly 4 bowlers?

4

16. **Case study based question:**

Given picture is showing Sreeja's home and school. Her school is at the position A and home is at the position B. assuming O is the origin, OA is along x-axis and OB is along y-axis. Answer the following questions.



i) What would be the co-ordinate of the point A and B?

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- (a) (3, 0), (2, 0)
- (b) (3, 0), (0, 2)
- (c) (0, 3), (0, 2)
- (d) (0, 3), (2, 0)

ii) What would be the equation of the line passing through A and B?

- (a) $2x + 3y = 6$
- (b) $3x + 2y = 6$
- (c) $2x - 3y = 6$
- (d) $3x - 2y = 6$

iii) A shopping mall is situated at the position C. If the slope of the line BC is 1, then its equation will be

- (a) $x + y = 2$
- (b) $x - y + 2 = 0$
- (c) $x + y = 0$
- (d) $x = y$

iv) What would be the distance of the origin from the line BC?

- (a) 1 km
- (b) 1.2 km
- (c) 1.4 km
- (d) 2 km

17. a. In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that

- (i) The student opted NCC or NSS.
- (ii) The student has opted for neither NCC nor NSS

Or

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- b.** The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Hindi examination?

Section – D

18. **a.** If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$, verify that
 (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

Or

6

- b.** In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products. Find how many liked product C only.

19. **a.** Find the sum to n terms of the series $5 + 55 + 555 + \dots$

Or

6

- b.** If A and G be A.M and G.M respectively between two positive numbers. Prove that the numbers are $A \pm \sqrt{(A + G)(A - G)}$.

20. **a.** Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $4x^2 + 9y^2 = 36$

Or

6

- b.** Find the equation of the circle which passes through the points $(2, 3)$ and $(-1, 1)$ and whose centre is on the line $x - 3y - 11 = 0$.

21. **a.** Find the mean deviation about the mean for the data

Income per day in ₹	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800
Number of persons	4	8	9	10	7	5	4	3

Or

6

- b.** Calculate the standard deviation and mean diameter of the circles:

Diameters	33-36	37-40	41-44	45-48	49-52
No. of circles	15	17	21	22	25
