2021
MATHEMATICS

Full marks: 80 Time: 3 hours

General instructions:
  i)   *Approximately 15 minutes is allotted to read the question paper and revise the answers.*
  ii) *The question paper consists of 21 questions. All questions are compulsory.*
  iii) *Marks are indicated against each question.*
  iv) *Internal choice has been provided in some questions.*
  v)   *Use of simple calculators (non-scientific and non-programmable) only is permitted.*
N.B: Check that all pages of the question paper is complete as indicated on the top left side.

Section – A

1. **Choose the correct answer from the given alternatives:**

(a) If $A = \phi$, then $n[P(A)] =$ **1**
   (i) 2 (ii) 3 (iii) 4 (iv) 1

(b) The value of $\sin 765°$ is equal to **1**
   (i) $\dfrac{1}{\sqrt{3}}$ (ii) $\dfrac{-1}{\sqrt{3}}$ (iii) $\dfrac{1}{\sqrt{2}}$ (iv) $\dfrac{-1}{\sqrt{2}}$

(c) The value of $i^9 + i^{19}$ is equal to **1**
   (i) -2 (ii) 0 (iii) 1 (iv) 2

(d) The number of permutations of n objects where p objects are of the same kind and rest are all different is equal to **1**
   (i) $\dfrac{p!}{n!}$ (ii) $\dfrac{(n-1)!}{p!}$ (iii) $\dfrac{n!}{p!}$ (iv) $n!$

(e) If $\dfrac{1}{8!} + \dfrac{1}{9!} = \dfrac{x}{10!}$, then the value of x is **1**
   (i) 5 (ii) 10 (iii) 50 (iv) 100

(f) Which term of the sequence $2,\ 2\sqrt{2},\ 4,\ ...$ is 128? **1**
   (i) 9th term (ii) 11th term (iii) 12th term (iv) 13th term

(g) The angle between the x-axis & the line joining the points $(3, -1)$ and $(4, -2)$ is **1**
   (i) 30° (ii) 45° (iii) 60° (iv) 135°

(h) $\lim_{x \to 0} \dfrac{\sin ax}{bx}$,$a,b \ne 0$ is equal to   **1**

   (i) $\dfrac{b}{a}$ (ii) $\dfrac{a}{b}$ (iii) ab (iv) 1

Section – B

2. If X and Y are two sets such that $X \cup Y$ has 18 elements, X has 8 elements and Y has 15 elements; how many elements does $X \cap Y$ have?   **2**

3. Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of 22 cm. (Use $\pi = \dfrac{22}{7}$)   **2**

4. Find the multiplicative inverse of the complex number $4 - 3i$ **2**

5. Solve the inequality: $3(x - 1) \le 2(x - 3)$ **2**

6. To receive grade 'A' in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Sunita's marks in first four examinations are 87, 92, 94 and 95, find minimum marks that Sunita must obtain in fifth examination to get grade 'A' in the course.   **2**

7. From a committee of 8 persons, in how many ways can we choose a chairman and a vice chairman assuming one person can not hold more than one position?   **2**

8. Find the ratio in which the YZ-plane divides the line segment formed by joining the points (−2, 4, 7) and (3, −5, 8).   **2**

9. Evaluate: $\lim_{x \to 0} \dfrac{\cos 2x - 1}{\cos x - 1}$ **2**

Section – C

10. Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that :
    (i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$ (ii) $A \times C$ is a subset of $B \times D$ **4**

11. **a.** Prove that: $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

    <div align="center">**Or**</div>   **4**

    b. Prove that: $\tan 4x = \dfrac{4 \tan x \left(1 - \tan^2 x\right)}{1 - 6 \tan^2 x + \tan^4 x}$

12.   Convert the complex number $\sqrt{3} + i$ into polar form.   **4**

13. **a.** In how many ways can the letters of the word PERMUTATIONS be arranged if the: (i) words start with P and end with S, (ii) vowels are all together?

<div align="center">**Or**</div>   **4**

   b. Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each other.

14. **a.** The sum of first three terms of a G.P. is $\dfrac{39}{10}$ and their product is 1. Find the common ratio and the terms.

<div align="center">**Or**</div>   **4**

   b. The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Determine the first term, the common ratio and the sum to n terms of the G.P.

15. Find the equation of the lines which cut off intercepts on the axes whose sum and product are 1 and −6 respectively.   **4**

16. **a.** Find the derivative of $\cos x$ from the first principle.

<div align="center">**Or**</div>   **4**

   b. Find the derivative to $\dfrac{4x + 5\sin x}{3x + 7\cos x}$

17.   Three coins are tossed once. Let A denote the event " three head show", B denote the event "two heads and one tail show", C denote the event " three tails show" and D denote the event "a head shows on the first coin". Which event are (i) simple (ii) compound?   **4**

<div align="center">**Section – D**</div>

18. **a.** If U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {2, 4, 6, 8}, B = {2, 3, 5, 7} and C = {7, 8, 9}, verify that: (i) $(A \cup B)' = A' \cap B'$,   (ii) $(A \cap B)' = A' \cup B'$, (iii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

<div align="center">**Or**</div>   **6**

   b. In a group of athletic teams of a public school in New Delhi, 18 are in basketball team, 25 are in the cricket team and 22 are in the football team. If 9 play basketball and cricket, 10 play cricket and football and 8 play football and basketball, find: (i) the total number of players in the group, (ii) the number of players who play cricket only, (iii) the number of players who play football only, given that 4 players of the group play all the three games.

19. **a.** Find the sum to n terms of the sequence 8, 88, 888, 8888, …
<div align="center">**Or**</div>   **6**
    b. Find four numbers forming a geometric progression in which the third term is greater than the first term by 9, and the second term is greater than the 4th by 18.

20. **a.** Find the equation of the circle passing through the points (4, 1) and (6, 5) and whose centre is on the line $4x+ y= 16$.
<div align="center">**Or**</div>   **6**
    b. Find the coordinates of the foci, the vertices, the lengths of the major axis and the minor axis, the eccentricity and the length of the latus rectum of the ellipse:

$$\frac{x^2}{4} + \frac{y^2}{25} = 1$$

21. **a.** Calculate the mean deviation about median age for the age distribution of 100 persons given below:

Age (in years)	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55
Number	5	6	12	14	26	12	16	9

<div align="center">**Or**</div>   **6**
    b. Find the mean and variance for the following frequency distribution:

Classes	0-10	10-20	20-30	30-40	40-50
Frequencies	5	8	15	16	6

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